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ExCAPE Meeting \\
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Fixed-point implementations of controllers are widely used in automotive and avionics industries.

\[ \dot{\xi} = f(\xi, \nu), \quad \xi(0) = \xi_0 \]

\[ \nu(t) = k(\xi(t)) \]

\( \xi \) - State of the plant

\( \nu \) - Control signal generated by the controller
Motivation

- Fixed-point implementations of controllers are widely used in automotive and avionics industries.
- Error is introduced in the implementation due to the use of finite-precision arithmetic.

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Fixed-point implementations of controllers are widely used in automotive and avionics industries.

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Performance of a controller is inversely proportional to the error introduced in the implementation [AntaMajumdarSTabuada EMSOFT 2010].

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**Objective:** To minimize error in the controller implementation.
Problem with Example: Batch Reactor Process

\[
\text{out} = (-0.0078) \times \text{state1} + 0.9052 \times \text{state2} +
(-0.0181) \times \text{state3} + (-0.0392) \times \text{state4} +
(-0.0003) \times y1 + 0.0020 \times y2
\]
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\]

Best Fixed-Point Program (16 bits)

\[
\text{tmp0} = ((-32716 \times \text{state1}) \gg 18) \\
\text{tmp1} = ((29662 \times \text{state2}) \gg 15) \\
\text{tmp2} = ((\text{tmp0} + (\text{tmp1} \ll 4)) \gg 4) \\
\text{tmp3} = ((-18979 \times \text{state3}) \gg 16) \\
\text{tmp4} = (((\text{tmp2} \ll 4) + \text{tmp3}) \gg 4) \\
\text{tmp5} = ((-20552 \times \text{state4}) \gg 15) \\
\text{tmp6} = (((\text{tmp4} \ll 4) + \text{tmp5}) \gg 4) \\
\text{tmp7} = ((-20133 \times y1) \gg 22) \\
\text{tmp8} = (((\text{tmp6} \ll 4) + \text{tmp7}) \gg 4) \\
\text{tmp9} = ((16777 \times y2) \gg 19) \\
\text{out} = (((\text{tmp8} \ll 4) + \text{tmp9}) \gg 4) \\
\text{return out}
\]
Problem with Example: Batch Reactor Process

\[ \text{out } = (-0.0078) \times \text{state1} + 0.9052 \times \text{state2} + \\
(-0.0181) \times \text{state3} + (-0.0392) \times \text{state4} + \\
(-0.0003) \times \text{y1} + 0.0020 \times \text{y2} \]

Error bound in the best fixed-point implementation (16 bits): 3.9e-03
Problem with Example: Batch Reactor Process

\[
\text{out} = (-0.0078) \times \text{state1} + 0.9052 \times \text{state2} + \\
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(-0.0003) \times y1 + 0.0020 \times y2
\]

Error bound in the best fixed-point implementation (16 bits): 3.9e-03

\[
\text{out} = ((0.9052 \times \text{state2}) + (((\text{state3} \times -0.0181) + \\
(-0.0078 \times \text{state1})) + (((-0.0392 \times \text{state4}) + \\
(-0.0003 \times y1)) + (0.002 \times y2))))
\]
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Error bound in the best fixed-point implementation (16 bits): 1.39e-03
Problem with Example: Batch Reactor Process

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\text{out} = (-0.0078) \times \text{state1} + 0.9052 \times \text{state2} +
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(-0.0003) \times \text{y1} + 0.0020 \times \text{y2}
\]

Error bound in the best fixed-point implementation (16 bits): 3.9e-03

\[
\text{out} = ((0.9052 \times \text{state2} ) + (((\text{state3} \times -0.0181)+
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(-0.0003 \times \text{y1} )) + (0.002 \times \text{y2 })))
\]

Error bound in the best fixed-point implementation (16 bits): 1.39e-03

Improvement 55%, without requiring any extra hardware
Problem with Example: Batch Reactor Process

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**Question:** How to synthesize the best expression automatically?
Problem with Example: Batch Reactor Process

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Error bound in the best fixed-point implementation (16 bits): 1.39e-03

Improvement 55%, without requiring any extra hardware

**Question:** How to synthesize the best expression automatically?

The problem is **NP-hard**
Apply genetic programming based search for optimal expression for a chosen controller

For each expression, the objective function captures the error bound in the best fixed-point implementation

Mutation and Crossover functions are defined on the AST of the controller expression

Bound on the error is computed using affine arithmetic
We compare the performance of our synthesized controllers with the controllers presented in [MajumdarSZamani EMSOFT 2012]

<table>
<thead>
<tr>
<th>Systems</th>
<th>Improvement (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>bicycle</td>
<td>10.96</td>
</tr>
<tr>
<td>dc motor</td>
<td>40.24</td>
</tr>
<tr>
<td>pitch angle</td>
<td>52.32</td>
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<tr>
<td>pendulum</td>
<td>19.26</td>
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<tr>
<td>batch reactor</td>
<td>20.08</td>
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Thank You!!