Uniformity and Counting

- Given a propositional logic formula $F$ in CNF
  - $(x \lor y \lor \neg z) \land (\neg x \lor \neg w \lor y) \land (\neg y \lor z \lor w \lor s)$

- Generate satisfying assignments uniformly at random
  - Uniform over the space of satisfying assignments

- Count number of satisfying assignments
Uniformity

- $F = (a \lor b)$

- $R_F: \{(0,1), (1,0), (1,1)\}$

- $G$: A uniform generator

- $Pr[G(F) = (0, 1)] = Pr[G(F) = (1, 1)] = \ldots = 1/3$
Counting

- $F = (a \lor b)$
- $R_F: \{(0,1), (1,0), (1,1)\}$
- $\#F: 3$
- $\#P$: The class of decision problems for counting problems in NP
Design Methodology

Computational Engines

Challenge Problems

Education and Knowledge Transfer

Tools and Evaluation

Constrained Random Simulation

Sketch based Synthesis

Sampling Techniques

Computational Engines

Automatic Problem Generation

Education and Knowledge Transfer

Apps for Multicore Engines

Multicore Protocols

Networked Systems

Robotic Systems

Challenge Problems
Diverse Applications

Constrained Random Simulation

Sketch based Synthesis

Sampling Techniques

Probabilistic Inference

Planning under uncertainty

Automatic Problem Generation
Central Idea
Partitioning into equal “small” cells
Partitioning into equal “small” cells

Pick a random cell

Pick a random solution from this cell
How to Partition?

How to partition into roughly equal small cells of solutions without knowing the distribution of solutions?

r-Universal family of hash functions
(Carter-Wegman 1979)
Counting through Partitioning

Pick a random cell

Total # of solutions = #solutions in the cell * total # of cells
Strong Theoretical Results

- UniGen: Scalable Uniform Generator

For every solution $y$ of $R_F$

\[
\frac{1}{2.7} \times \frac{1}{|R_F|} \leq \Pr [y \text{ is output}] \leq 2.7 \times \frac{1}{|R_F|}
\]

- Scales to hundreds of thousands of variables
Results: Uniformity

- Benchmark: case110.cnf; #var: 287; #clauses: 1263
- Total Runs: $4 \times 10^6$; Total Solutions: 16384
Strong Theoretical Results

ApproxMC (CNF: F, tolerance: \( \varepsilon \), confidence: \( \delta \))

Suppose \( \text{ApproxMC}(F, \varepsilon, \delta) \) returns \( C \). Then,

\[
\Pr \left[ \frac{\#F}{1+\varepsilon} \leq C \leq (1+\varepsilon) \#F \right] \geq \delta
\]
Can Solve a Large Class of Problems

Large class of problems that lie beyond the exact counters but can be computed by ApproxMC.
Summary

- Approximate counting and uniform generation: theoretical and practical interest (diverse applications)

- Prior work didn’t provide scalability along with guarantees

- Provides strong theoretical guarantees

- Scales to formulas involving hundreds of thousands of variables
Publications

