Partial-observation Stochastic Parity Games

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Parity Games

**Canonical model** for reactive synthesis

- Two-player finite-state infinite-duration games
- Objectives: $\omega$-regular objectives specified as parity conditions
- Analysis:
  - In $\text{NP} \cap \text{co-NP}$
  - Precise complexity: *major open problem*
  - Good algorithms: *major open problem*
Stochastic Parity Games

- Two-player parity games with *stochastic* transitions
  - Player 1 and player 2: *strategic*
  - Transitions: *stochastic*.

- Goal: Synthesis of stochastic reactive systems
  - Actions have *uncertain* outcomes
Basic Questions

- **Qualitative analysis:** Does there exist a player-1 strategy to ensure the parity objective with probability 1 (almost-sure winning) against all strategies of player 2? [Similar question for positive (>0) winning] – *This talk!*

- **Quantitative analysis:** *Does there exist a player-1 strategy to ensure the parity objective with probability at least \( \lambda \), for \( \lambda \in (0,1) \), against all strategies of player 2?*
Qualitative Analysis of Stochastic Parity Games

- **Turn-based games**: \( NP \cap co-NP \) [Chatterjee-Jurdziński-Henzinger, 2004]
Complete vs Incomplete Information

- Full-observation stochastic games
  - Both players have *perfect* information about the history of the game.

- Partial-observation stochastic games
  - One or both players have *partial* information about the history of the game.

- *Motivation*: synthesis from components
Partial-Observation Stochastic Parity Games

- **Undecidable** [Baier-Bertrand-Größer, 2008]
  - Even with a single player (POMDP)
Finite-Memory Stochastic Parity Games

- **Motivation**: synthesis – *finite-memory strategy*

- **Problem**: Stochastic parity game
  - Player 1: partial observation
  - Player 2: full observation (powerful adversary)
  - Question: Can player 1 win almost surely using a *finite-memory strategy*?

- [Nain-V., 2013]: Decidable in 2EXPTIME
- [CDNV, 2014]: EXPTIME-complete
Key Ideas

- Under finite-memory strategies, precise probabilities do not matter.

- Analysis of recurrent classes in the Markov chains given the strategies.

- In [NV13] reduction to emptiness of alternating tree automata.
  - Exponential reduction to enumerate recurrent classes.

- In [CDNV14] polynomial reduction to alternating tree automata
  - Local gadgets: reduce from 2-player stochastic game to 3-player strategic game
Conclusion

- Models of stochastic games for synthesis of stochastic reactive systems.

- Partial-observation stochastic parity games: old *negative* results and recent *positive* results.