Robust Synthesis for Reachability Specifications with Unmodeled Disturbances

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Motivation

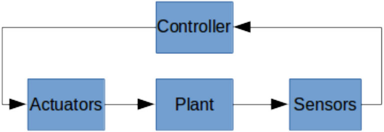

Figure : A Control Problem

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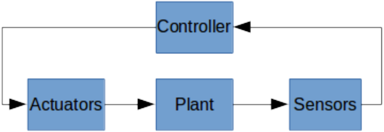

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- What if the actuators/sensors don't always work?

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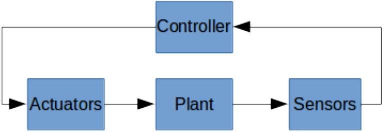


Figure : A Control Problem

- What if the actuators/sensors don't always work?
  - Intermittent Faults (Mechanical, Communication, etc...)
  - Unmodeled Disturbances

Model

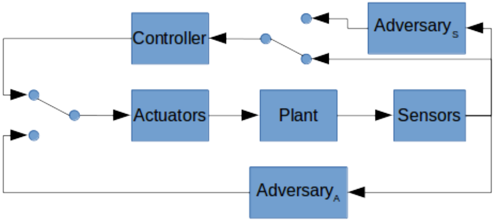
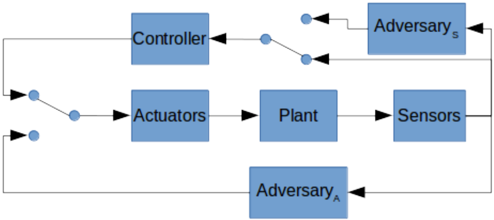


Figure : A Control Problem with Intermittent Unmodeled Disturbances

Model

Normal Operation:



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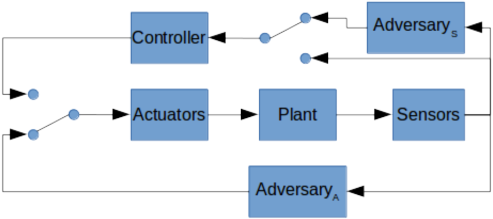
Abnormal Operation:



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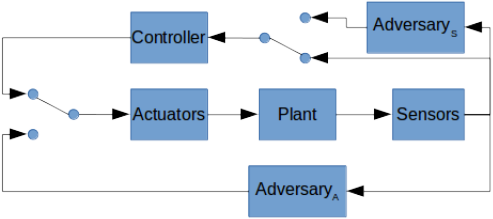

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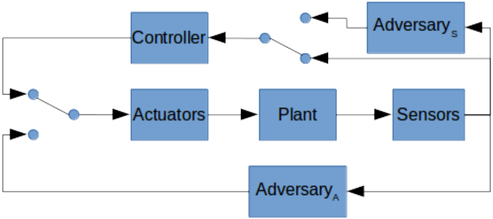


Figure : A Control Problem with Intermittent Unmodeled Disturbances

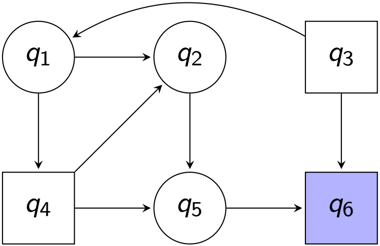
Assumption:

- The effect of unmodeled disturbances is equivalent to a change of control inputs

Definition (Reachability Game)

$\mathcal{R} = (V_0, V_1, E, F)$, where F is the target set for Player 0.

Example:



$$V_0 = \{q_1, q_2, q_5\}$$
$$V_1 = \{q_3, q_4, q_6\}$$
$$F = \{q_6\}$$
$$E = \dots$$

Player 0 = Controller, Player 1 = Adversary

Reachability Games with Unmodeled Disturbances

Definition (Reachability Game with Unmodeled Disturbances)

$\mathcal{R} = (V_0, V_1, E, F, D)$, where $D \subseteq E \cap (V_0 \times V)$ defines the "capabilities" of unmodeled disturbances.

Interpretation:

- Unmodeled disturbances are taken to be adversarial

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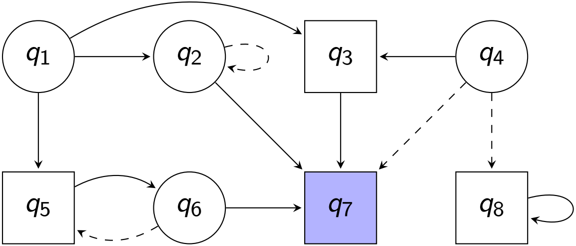
- Unmodeled disturbances have the same effect as a as change of control $\Rightarrow$ Player 1 may take control of Player 0's turn and choose the next vertex
- No information about unmodeled disturbances $\Rightarrow$ $D(v) = E(v)$
- Unmodeled disturbances can't occur from v $\Rightarrow$ $D(v) = \emptyset$

Reachability Games with Unmodeled Disturbances

Example:



$$V_0 = \{q_1, q_2, q_4, q_6\} \qquad V_1 = \{q_3, q_5, q_7, q_8\}$$

$$F = \{q_7\} \qquad E = \dots \qquad D = \{(q_2, q_2), (q_4, q_7), (q_4, q_8), (q_6, q_5)\}$$

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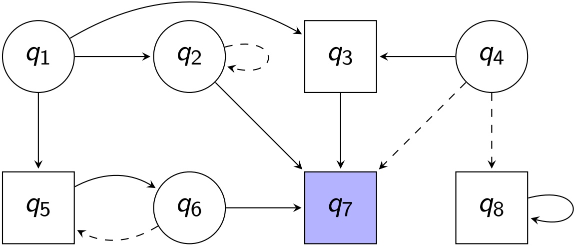
$$\mu(v) := \begin{cases} 0 & \text{if } v \in W_1; \\ \min_{v' \in E(v)}\{\mu(v')\} & \text{if } v \in W_0 \cap V_1; \\ \min\{\max_{v' \in E(v)}\{\mu(v')\}, & \\ \quad 1 + \min_{v' \in D(v)}\{\mu(v')\}\} & \text{if } v \in W_0 \cap V_0. \end{cases} \tag{1}$$

Computing μ

Example:



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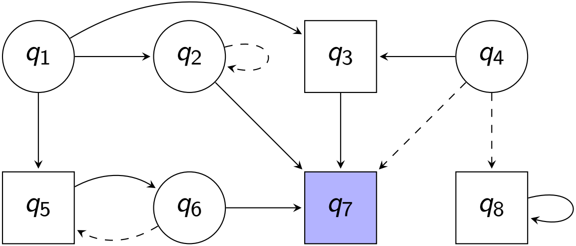

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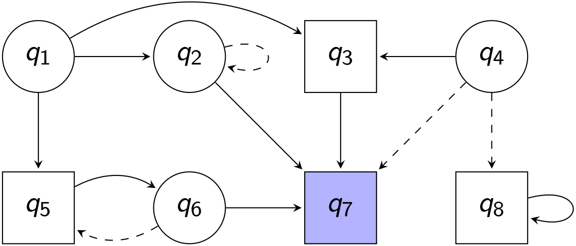

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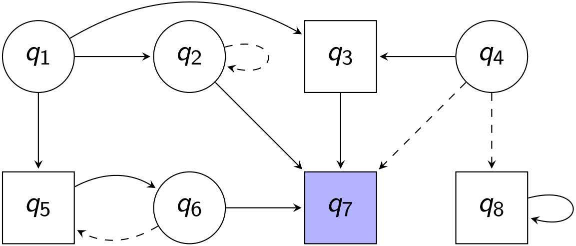

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$$\mu(q_2) = \mu(q_5) = \mu(q_6) = ?$$

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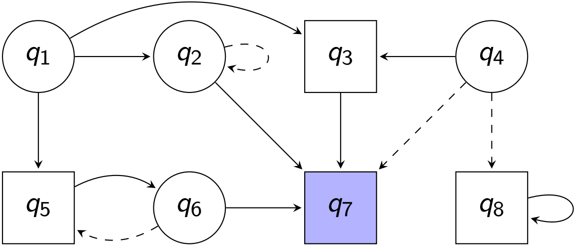

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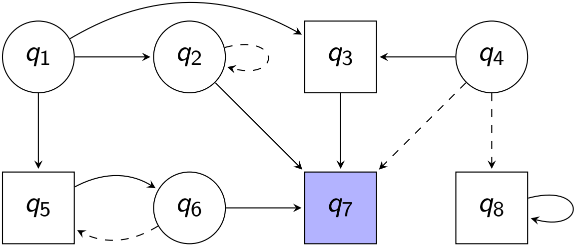
The Three Cases



- Guaranteed Player 0 victory (e.g., q_1, q_3, q_7)
- Player 1 victory with finite number of unmodeled disturbances (e.g., q_4, q_8)
- Other vertices: Look at the *proportion* of turns consisting of unmodeled disturbances (e.g., q_2, q_5, q_6)

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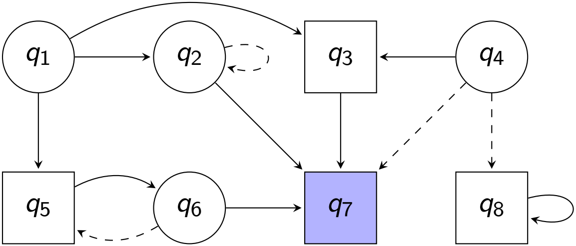

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- Other vertices: Look at the *proportion* of turns consisting of unmodeled disturbances (e.g., q_2, q_5, q_6)
- $\Rightarrow \overline{\mu}(q_2) = 1$, $\overline{\mu}(q_5) = \overline{\mu}(q_6) = 1/2$

Weighted Mean Payoff Games

Definition (Weighted Mean Payoff Game)

$\mathcal{M} = (V_0, V_1, E, r, w)$, where $r : E \to \mathbb{R}$ is a reward function and $w : E \to \mathbb{R}$ is a weight function.

Player 0's objective: Maximize

$$\liminf_{n\to\infty} \frac{\sum_{i=0}^{n} w((v_i, v_{i+1}))r((v_i, v_{i+1}))}{\sum_{i=0}^{n} w((v_i, v_{i+1}))} \tag{2}$$

over the set of runs $v_0 v_1 v_2 \ldots$.

The Mapping

- $v \in F \Rightarrow$ Remove all outgoing edges and place self-loop with $r = 1$, $w = 1$.
- $v \in V_1 \setminus F \Rightarrow$ One edge for each $v' \in D(v)$ with $r = 0$, $w = 1$.
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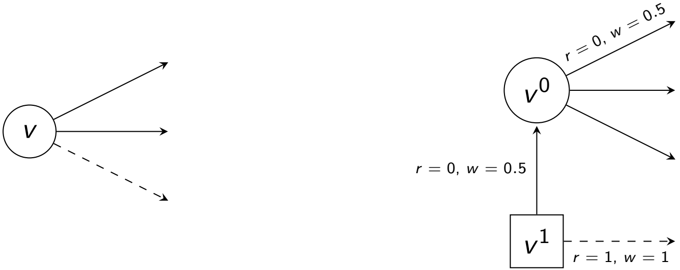

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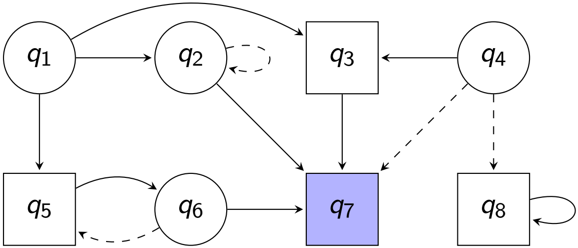

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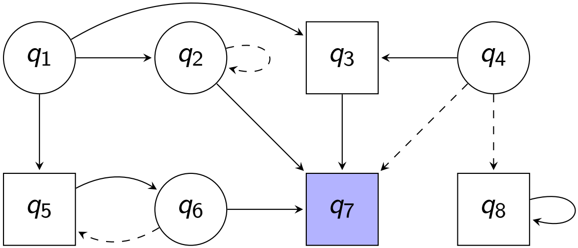

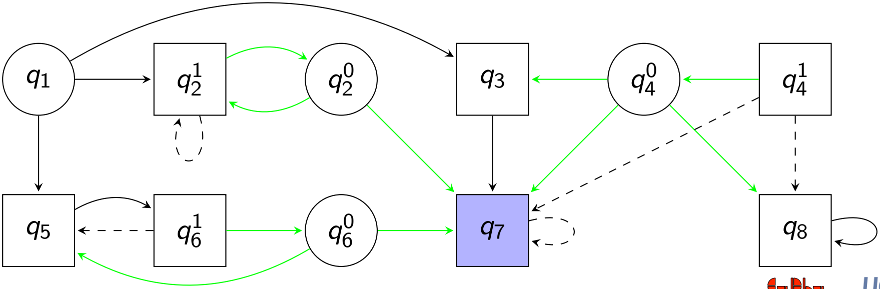

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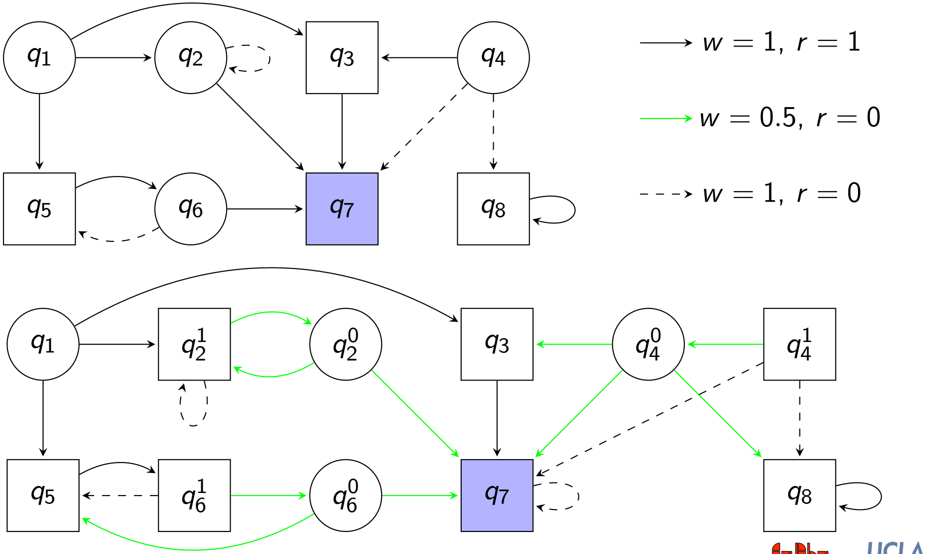

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4. Mean payoffs $\{\nu_k(v)\}_{v \in V}$ of the k step game converge to values $\{\nu(v)\}_{v \in V}$ of the infinite duration game
5. For k large enough, there exists only a single feasible value for $\nu(v)$ that exists within the error bounds around $\nu_k(v)$.

Solving the Weighted Mean Payoff Game

- Solve for game values $\{\nu_k(v)\}_{v \in V}$ of the $k = 4|V|^3$ step game (Time: $O\left(|V|^3|E|\right)$)
- True game values are only rationals of the form $\frac{a}{b}$ with $b \leq |V| := n$ in
$$\left(\nu_k(v) - \frac{1}{2n(n-1)}, \nu_k(v) + \frac{1}{2n(n-1)}\right)$$
- Synthesize optimal strategy by binary search:
  1. Pick a vertex with multiple outgoing edges. Eliminate half.
  2. Recompute game values and compare.
  3. Same $\Rightarrow$ There exists an optimal strategy using only the remaining edges. Different $\Rightarrow$ Replace remaining edges with removed ones.
  4. Repeat until every vertex has outdegree 1.
- Time: $O\left(|V|^4|E|\log(|E|/|V|)\right)$

Obtaining the Final Strategy

Recall three cases:

1. Guaranteed Player 0 victory $\overline{W}_0$
2. Player 1 victory with finite number of unmodeled disturbances $\{v \in V : \mu(v) < \infty\}$
3. Other vertices $\{v \in V : \mu(v) = \infty\} \setminus \overline{W}_0$

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Final Strategy: $f_0 : V_0 \to V$

- $v \in \overline{W}_0 \Rightarrow f_0(v) \in \overline{W}_0$
- $\mu(v) < \infty \Rightarrow f_0(v) \in \text{argmax}_{v' \in E(v)}\, \mu(v')$
- $\mu(v) = \infty \wedge v \notin \overline{W}_0 \Rightarrow f_0(v)$ as computed from weighted MPG

Conclusion

- Showed how to synthesize controllers robust to unmodeled disturbances for safety (past work) and reachability specifications.
- Future work: experimental validation.